NEW AND OLD PROOFS OF THE PYTHAGOREAN THEOREM.

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[Continued from March Number.]

The following "dissection proofs" differ from the preceding ones only in the fact that there is a displacement of one or more of the squares from their usual places.

LXXXIII. Fig. 34.
Let the $\triangle ABC$ be right-angled at C.
$\triangle LHK = \triangle ABC$, and $\triangle LAE = \triangle HBF$.
$\therefore ABHL \backsimeq ACDE + DFHK.$ Q. E. D.

LXXXIV. Fig. 34.
$LMOA \backsimeq LKCA \backsimeq ACDE.$
$HMOB \backsimeq HKCB \backsimeq HKDF.$
$\therefore ABHL \backsimeq ACDE + DFHK.$ Q. E. D.

LXXXV. Fig. 34.
$ANQB \backsimeq AEFB \backsimeq AEDC.$
$LNQH \backsimeq LEFH \backsimeq FHKD.$
$\therefore ABHL \backsimeq ACDE + DFHK.$ Q. E. D.

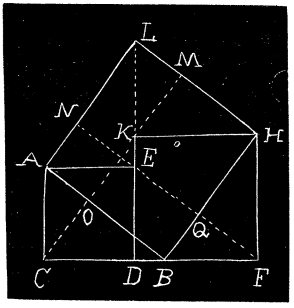

Fig. 34.

NOTE. In Fig. 34, if ALE is taken as the triangle, we have another type of figure, thus giving several more proofs. See Halsted's *Elements of Geometry*, page 78.

LXXXVI. Fig. 35.
The given triangle is ABC.
$\triangle LNO = \triangle APE'$. $\triangle LMO = \triangle ABC$.
$\triangle AHM = \triangle MDA \backsimeq PADE + BNK.$
$\therefore AMLB \backsimeq ADEE' + AHKC.$ Q. E. D.

LXXXVII. Fig. 35.
$LRSB \backsimeq LOCB \backsimeq ADEE'.$
$AMRS \backsimeq AMOC \backsimeq AHKC.$
$\therefore AMLB \backsimeq ADEE' + AHKC.$ Q. E. D.

LXXXVIII. Fig. 35.
$ABTX \backsimeq ABVD \backsimeq ADEE'.$
$MLTX \backsimeq MLVD \backsimeq AHKC.$
$\therefore AMLB \backsimeq ADEE' + AHKC.$ Q. E. D.

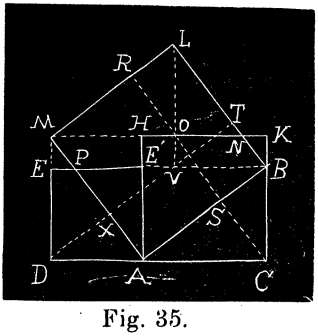

Fig. 35.

NOTE. In Fig. 35, if AMH, MLO, or LBV is taken as the given triangle, we have slightly different types of figures, eaeh yielding various proofs.

LXXXIX. Fig. 36.
$AOC=APE$. $MNB=HFB$.
$MNOL=HKPB$.
$\therefore ABML=ACDE+DFHK$. Q. E. D.

XC. Fig. 37.
$PBR=SBK$. $RPLO=ETAD$.
$LNO=ATF$. $NMAC=SBAH$.
$\therefore ABLM=ADEF+AHKC$. Q. E. D.

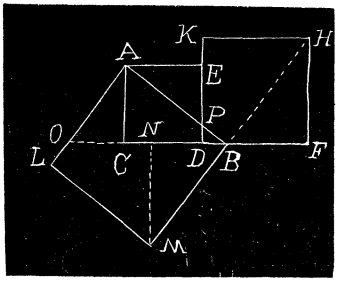

Fig. 36.

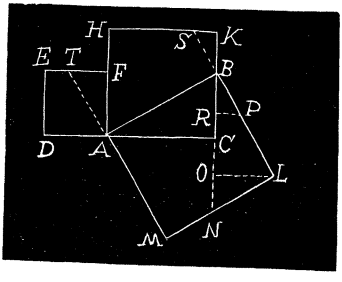

Fig. 37.

It will be observed that the above proofs differ chiefly in the relative position of the triangle and the squares. In order to give an idea of still other possible varieties, it may be noted for example, that instead of the arrangement as we have it it Fig. 35, the given triangle may have seven other positions within the square $ACKH$, right angles coinciding. Furthermore, in each case, the square on the hypotenuse may be constructed outwardly from the triangle, or overlapping it. One of these cases, it will be remembered, was considered in November number, 1897.

[To be continued.]

DEPARTMENTS.

SOLUTIONS OF PROBLEMS.

ARITHMETIC.

104. Proposed by ALOIS F. KOVARIK, Instructor in Mathematics and Physics, Decorah Institute, Decorah, Iowa.

If I should buy goods at a price 20% higher than I did buy them, and sell the goods for the same amount that I did sell them, I would gain 25% less than I did gain. What per cent. did I gain? (Solve by Arithmetic).

I. Solution by W. F. DRADBURY, Head Master, Cambridge Latin School, Cambridge, Mass.

If I gain 25% less, I get $\frac{6}{5}\times\frac{25}{100}$, or $\frac{150}{500}$ of \$1 less than if I sold at the same per cent. advance. If I sell at same per cent. advance, I should receive $\frac{6}{5}(1+\frac{r}{100})=\frac{6}{5}+\frac{6r}{500}$. What I did receive was $1+\frac{r}{100}$. Subtract, and we have